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# The distribution of productivity: Measurement and implications

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**Empirical justification for the widespread use of the Pareto assumption is conflicted. The uncertainty surrounding productivity distributions arises from the use of sales to proxy for productivity and the inconsistency in fitting methodologies. In this paper, we use estimations of elasticities of substitution and firm-level productivity to directly assess its distribution and inherent gains from trade.**

## Overview

Melitz (2003) is typically augmented with the distributional assumption of **Pareto-productivity**. This assumption makes the model analytically tractable and results in a simple Gains From Trade (GFT)-equation

$$\hat{W} = \hat{\lambda}^{1/\hat{\epsilon}}$$

which can be estimated using a gravity model on macro data. The Pareto assumption thus makes the **micro structure redundant**.

Empirical evidence for this assumption is, however, inconsistent/inconclusive due to differences in

- Productivity measurement (Domestic/exporting/total sales?)
- Assumption on elasticity of substitution
- Distribution fitting & selection

**This paper** provides an

1. Estimation framework for both demand-side ($\sigma_s$) **and** supply-side ($\omega$) variables that is **widely applicable**
2. Formal framework to analyze the **tail** of the distribution
3. Evaluation of welfare implications for the **complete** economy

## Methodology

**Productivity measurement** The use of (domestic) sales as a proxy for productivity leaves no room for uncertainty, measurement error and heterogeneity in sales other than productivity and requires additional assumptions (f.i. on $\sigma_s$):

$$r(\omega) \sim h\left(r(\omega_{min}), \frac{k}{\sigma_s - 1}\right) \Leftrightarrow \omega \sim h\left(\omega_{min}, k\right).$$

We use the dynamic basis of the Melitz-model (De Loecker, 2011) to estimate

$$r_{ft} = \frac{p_{ft}y_{ft}}{P_{st}} = Y_{st}^{-\frac{1}{\sigma_s}} H_s^*\left(\boldsymbol{X}_{jt}\right) \omega_{ft}^* \epsilon_{ft}^*,$$

obtaining demand-side ($\sigma_s$) **and** supply-side ($\omega$) variables from **one** estimation procedure, while controlling for uncertainty and measurement error in output.

**Distribution fitting**: Maximum Likelihood with tail length determined from the minimum distance between fitted and empirical CDF.

**Distribution selection**: Likelihood ratio tests between Lognormal and (truncated) Pareto

## Data

Our dataset is extracted from the Amadeus database provided by Bureau Van Dijk. We withhold 11 European countries for 12 selected manufacturing industries, resulting in an analysis of **132 country-industry pairs** over the period 1996–2011.

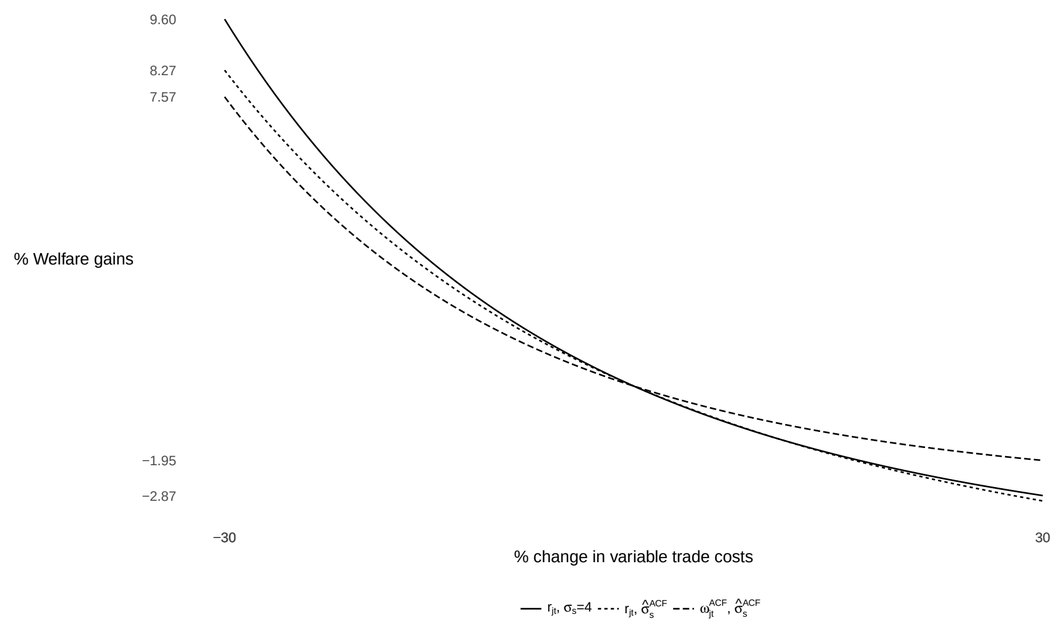
## Results

1. Truncated Pareto is a good fit for the tail!
2. Assuming a common distribution accross countries/industries is reasonable, but one needs to allow for heterogeneity in the parameters.
3. Using sales results in a more heterogeneous distribution than estimated productivity.

| | | % in tail | | Pareto shape | | |
|---|---|---|---|---|---|---|
| Industry | $\hat{\sigma}^{ACF}$ | $r_{ft}$ | $\omega_{ft}^{ACF}$ | $r_{ft}^{1/(4-1)}$ | $r_{ft}^{1/(\hat{\sigma}_s^{ACF}-1)}$ | $\omega_{ft}^{ACF}$ |
| 10t12 | 2.45 | 33.65 | 29.98 | 2.40 | 1.16 | 4.81 |
| 13t15 | 3.17 | 37.79 | 33.18 | 2.54 | 1.83 | 6.78 |
| 16 | 3.31 | 40.33 | 28.83 | 2.27 | 1.75 | 5.50 |
| 17t18 | 2.99 | 32.08 | 43.93 | 2.25 | 1.49 | 6.18 |
| 20 | 2.46 | 38.27 | 44.30 | 2.22 | 1.08 | 3.96 |
| 22 | 2.97 | 32.19 | 28.30 | 2.89 | 1.89 | 6.17 |
| 23 | 4.77 | 31.37 | 39.90 | 2.23 | 2.80 | 5.79 |
| 24t25 | 3.16 | 33.22 | 37.61 | 2.20 | 1.58 | 6.77 |
| 26t27 | 2.56 | 38.74 | 30.60 | 2.07 | 1.08 | 5.20 |
| 28 | 3.31 | 31.21 | 30.58 | 2.29 | 1.76 | 4.87 |
| 29t30 | 3.94 | 54.96 | 51.56 | 1.80 | 1.76 | 6.28 |
| 31 | 4.64 | 44.99 | 32.74 | 2.30 | 2.78 | 7.19 |

## Welfare Implications

Calibration of multi-sector Melitz-model with composite Lognormal-Truncated Pareto distribution




## Take-Home

Our work highlights the possibilities and advantages of using firm-level productivity to assess aggregate gains from trade. A truncated Pareto provides a good fit to the tail of productivity distributions across countries and industries when one allows for heterogeneity in the distribution parameters. **Firm-level heterogeneous noise and wrong assumptions on the elasticities of substitution result in significant welfare differences.**