Returns to Scale & Aggregate Productivity

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Introduction

Motivation

- How can we understand rising returns to scale (RTS) and stagnating productivity?
- Growing evidence of rising returns to scale.
- Intuitively, recent technologies increase returns to scale:
  - intangible investment, IT and cloud infrastructure
- But as RTS rise in countries associated with these technologies, productivity is stagnating. This is puzzling...
- Typically higher RTS means higher productivity
  - True at the firm level
  - Unclear in aggregate due to competing channels e.g. selection, market power, allocation

What do we do? What do we find?

Empirical Contribution

- Estimate RTS in the UK economy.
- We find that firm-level RTS are increasing on average.

Theoretical Contribution

- How are productivity and RTS related?
- Firm dynamics model with imperfect competition and RTS.
  - Derive endogenous RTS and clarify sources of RTS.
  - Derive aggregate TFP and decompose
  - Show different sources of RTS have different implications
  - RTS in variable inputs seem better candidate than FC.
- Increases in RTS over the past couple of decades should've caused productivity to explode! **But**, if we allow markups to rise at the same time it erodes nearly all the productivity gains.

Returns to Scale

- Growing returns to scale:
  - Costs are less responsive to output.
  - Output is more responsive to inputs.
- **Cost side:** $\mathrm{RTS} = (d \ln \mathcal{C} / d \ln y)^{-1} = AC/MC$
- High RTS means costs unresponsive to output.
- Small firm has high RTS because fixed cost dominates, so change in output has little effect on costs.
- **Production side:** $\mathrm{RTS} = \nabla y(X) = \sum_{i=1}^{M} \frac{\partial \ln y}{\partial \ln x_i}$
- Output responds proportionally more for a small firm.

Returns to Scale Diagram

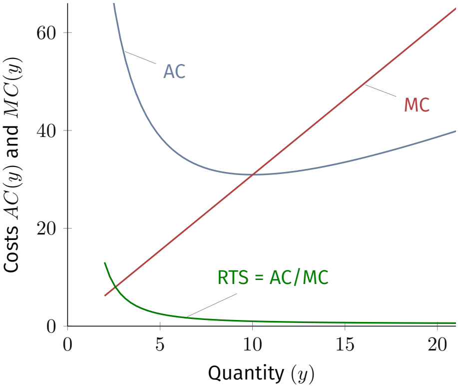


Figure 1 Fixed Cost with Increasing MC, U-Shaped AC Curve

Empirical Section

Data

- ARDx dataset is the UK's annual production survey.
- Runs from 1998 - 2014 and covers all sectors of the economy.
- 50,000 firms per year, 11m workers, 2/3 of GVA.
- All large firms ($>$250 employees) and a representative sample of smaller firms.
- We use data on: value added (revenue), labour (no. employees), materials and investment.
- We construct capital stock using the perpetual inventory method from firm-level investment data.

Returns to Scale Estimation

- We estimate Cobb-Douglas production functions:

$$\ln y_t(\imath) = \ln A_t(\imath) + \beta_1 \ln k_t(\imath) + \beta_2 \ln \ell_t(\imath) + \varepsilon_t(\imath).$$

- Endogeneity problem: cannot observe productivity $A_t(\imath)$ which affects optimal k_t, ℓ_t choices.
- Use Ackerberg, Caves, and Frazer (2015) and Gandhi, Navarro, and Rivers (2020) methodology.
- The sum of the coefficients is returns to scale (in variable inputs):

$$\beta_1 + \beta_2 = \nu.$$

- This measures the slope of the marginal cost curve.

Average Returns to Scale are Rising

- Pooling all firms over all years:
  - $\nu = 1.05$ and N = 527,813
- Pooling all firms over all years and studying sub-periods:

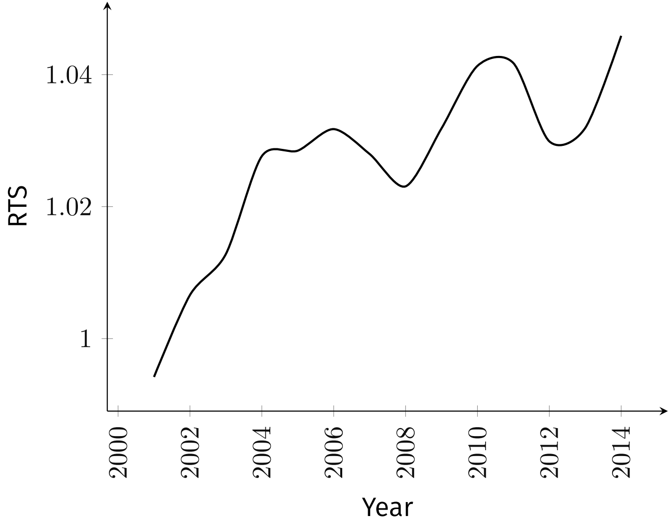
Table 1 Cobb-Douglas production function, 1998 - 2014, ACF

	1998 - 2001	2002 - 2005	2006 - 2009	2010 - 2014
ν	0.99	1.08	1.05	1.06
N	153,874	144,465	108,619	120,855

- At 2-digit industry vast majority experience increase.

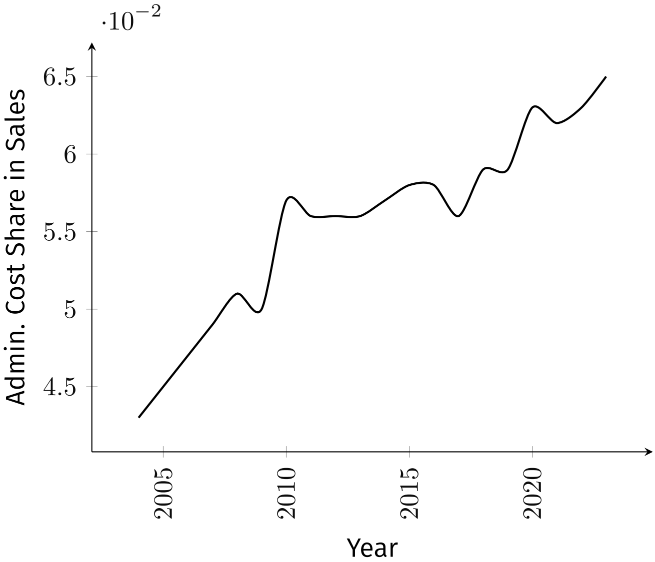
Rising Returns to Scale (rolling window GNR)

Figure 2 Returns to Scale in the UK, 2001 - 2014 using GNR



Fixed Cost Share is rising

Figure 3 Median Fixed Cost Share in Sales, BvD FAME



Model *(lite)*

Model Overview

- General equilibrium neoclassical growth with endogenous industry structure
- Monopolistic competition (fixed markups)
- Firm-level RTS
- Entry à la Hopenhayn (Pareto distributed firms)
- Productivity cut-off determining selection – how this moves is key.
- Household side standard
- Aggregate labour is fixed and exogenous.
- Firm side more involved...

Final Goods Producer

- Final goods producer solves

$$\Pi_t^F = \max_{y_t(\imath)} \quad Y_t - \int_0^{N_t} p_t(\imath) y_t(\imath) d\imath$$

$$\text{s.t.} \quad Y_t = N_t \left[\frac{1}{N_t} \int_0^{N_t} y_t(\imath)^{\frac{1}{\mu}} d\imath \right]^{\mu}$$

Markup

- The parameter $\mu \in (1, \infty)$ captures product substitutability.
- Optimality yields inverse-demand for firm:

$$p_t(\imath) = \left(\frac{Y_t}{N_t y_t(\imath)} \right)^{\frac{\mu - 1}{\mu}} .$$

- Hence there is downward-sloping demand.

Intermediate Goods Producer

Timeline

1. A firm pays cost κ to enter. Free entry holds.
2. Receives technology draw $A(\jmath)$ where $\jmath \in [0, 1]$ is uniform.
3. Given productivity draw, it decides whether to be active
   - Overhead cost ϕ causes some entrants to remain inactive.
4. All firms, both active and inactive, exit after one period.

Intermediate Goods Producer

- Firms have a fixed cost and sloping marginal costs

$$y_t(\jmath) = A(\jmath)\left[k_t(\jmath)^{\alpha}\ell_t(\jmath)^{1-\alpha}\right]^{\nu}$$
$$\ell_t(\jmath) = \ell_t^{tot}(\jmath) - \phi$$

- Therefore

$$y_t(\jmath) = A(\jmath)\left[k_t(\jmath)^{\alpha}[\ell_t^{tot}(\jmath) - \phi]^{1-\alpha}\right]^{\nu}$$

- Key parameters
  - ϕ fixed cost
  - ν RTS in variable inputs (MC slope) $\nu \uparrow \implies MC \downarrow$
- True RTS are a function of ϕ and ν
- Parameter ν captures *returns to scale in variable production*
  - $0 < \nu < 1$ decreasing returns in variable production (up-sloping MC)
  - $\nu = 1$ constant returns in variable production (flat MC)
  - $\nu > 1$ increasing returns in variable production (down-sloping MC)

Aggregate TFP

- $J_t \in (0,1)$ is productivity cut-off (i.e. the min prod. draw to cover your ϕ and make zero profits)
- E_t is measure of entrants $N_t = (1-J_t)E_t$
- Utilization u_t as the fraction of production labour in total labour:

$$u_t \equiv \frac{E_t \int_{J_t}^1 \ell(\jmath)d\jmath}{L_t} \qquad 1-u_t = \frac{N_t\phi}{L_t}.$$

- We can derive aggregate output

$$Y_t = TFP(J_t)K_t^{\alpha\nu}L_t^{1-\alpha\nu}$$

- TFP term captures aggregate productivity and can be decomposed into two terms $TFP_t = \Omega_t \times \hat{A}_t$:

$$TFP(J_t) \equiv \underbrace{\left(\frac{1-u_t}{\phi}\right)^{1-\nu} u_t^{(1-\alpha)\nu}}_{\text{allocative efficiency } \Omega_t} \underbrace{\left[\frac{1}{1-J_t}\int_{J_t}^1 A(\jmath)^{\frac{1}{\mu-\nu}}\, d\jmath\right]^{\mu-\nu}}_{\text{technical efficiency } \hat{A}_t}$$

- Alloc. eff. is all about fixed cost. Tech. eff. is selection.
- If $\phi = 0$ then $u_t = 1$ and $TFP_t = \hat{A}_t$

Model Analysis

Pareto Distribution

- Pareto distribution:

$$A(\jmath) = \frac{1}{(1-\jmath)^{1/\vartheta}}.$$

Pareto shape parameter

- $\vartheta > 1$ and as $\vartheta \to 1$ implies fatter right tail.
- An increase in J_t increases average productivity.
- The power mean, which appears in TFP, is:

$$\left[\frac{1}{1-J_t}\int_{J_t}^{1} A(\jmath)^{\frac{1}{\mu-\nu}}\, d\jmath\right]^{\mu-\nu} = \left(\frac{(\mu-\nu)\vartheta}{(\mu-\nu)\vartheta-1}\right)^{(\mu-\nu)} A(J_t)$$

- Technology term captures selection

Returns to Scale

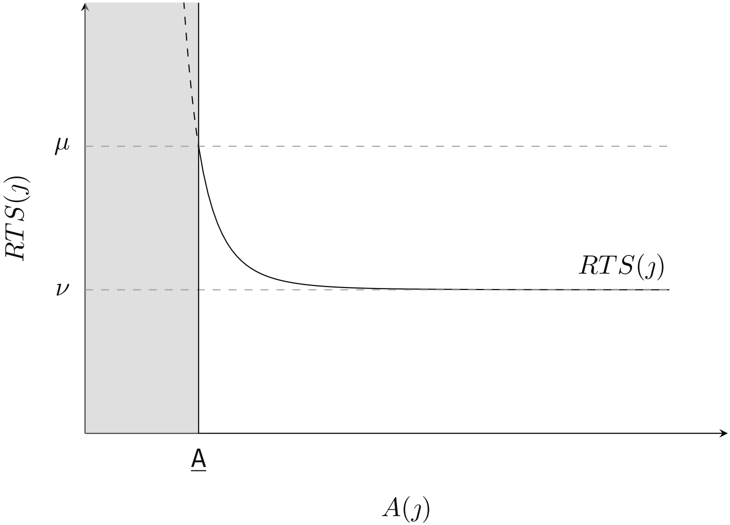
- Response of firm output to a change in *all* inputs:

$$\mathsf{RTS}_t(\jmath) \equiv \frac{\partial \ln y_t(\jmath)}{\partial \ln k_t(\jmath)} + \frac{\partial \ln y_t(\jmath)}{\partial \ln \ell_t^{tot}(\jmath)}$$
$$= \nu \left(1 + (1-\alpha)\frac{\phi}{\ell_t(\jmath)}\right)$$
$$= \nu + (\mu - \nu)\left[\frac{A(J_t)}{A(\jmath)}\right]^{\frac{1}{\mu-\nu}} .$$

- $\mathrm{RTS} \in (\nu, \mu)$ for high and low productivity draws.

Endogenous Returns to Scale

Figure 4 Firm-level Returns to Scale



Theoretical Results

- How do different sources of firm-level returns to scale (ν and ϕ) affect:
  1. Average returns to scale
  2. Average productivity

Average RTS Results

- Average firm under Pareto has

$$R\bar{T}S(J) = \nu + \frac{\vartheta(\mu-\nu)^2}{1+\vartheta(\mu-\nu)}$$

- Average returns to scale are increasing in span of control ν.

$$\frac{\partial R\bar{T}S(J)}{\partial \nu} = \frac{1}{(1+\vartheta(\mu-\nu))^2} > 0$$

- Average returns to scale are invariant to the fixed costs ϕ.

$$\frac{\partial R\bar{T}S(J)}{\partial \phi} = 0$$

- If ϕ increases:
  - All firms higher RTS.
  - But some high RTS firms become inactive.
  - Selection exactly offsets individual firm effect.

Aggregate Productivity

- Under Pareto model has a tractable steady state.
- *How do firm-level returns to scale affect aggregate productivity?*
  - Increase in ν
  - Increases in ϕ
- Aggregate productivity depends on allocative and technical efficiency

$$\ln TFP = \underbrace{\ln \Omega}_{alloc.} + \underbrace{\ln \hat{A}}_{tech.}$$

- Technical efficiency depends on selection.
- Allocative efficiency depends on the number of firms.

Change in ϕ

- Changes in fixed costs affect aggregate TFP through an allocation component and a technology component:

$$\frac{d\ln TFP}{d\ln\phi} = \frac{d\ln\Omega}{d\ln\phi} + \frac{d\ln\hat{A}}{d\ln\phi}$$

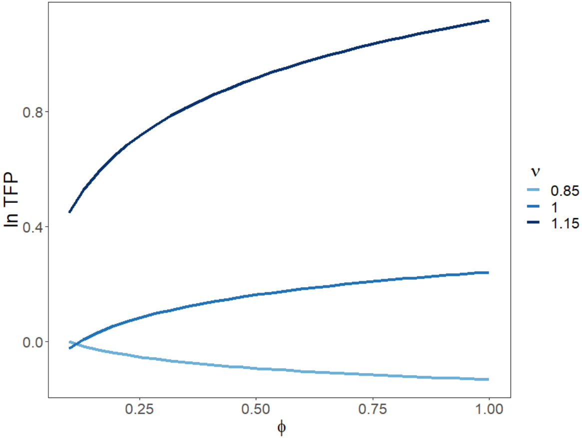
- Higher fixed costs increase selection which enhances productivity
- Allocation effect is ambiguous whether more or less firms is good for the division of resources depends on ν

$$\frac{d\ln TFP}{d\ln\phi} = -(1-\nu) + \frac{\nu(1-\alpha)}{\vartheta(1-\alpha\nu)-1}$$

- The effect is independent of μ

ϕ may raise productivity

Figure 5 Effect of ϕ on TFP for different ν



Change in ν

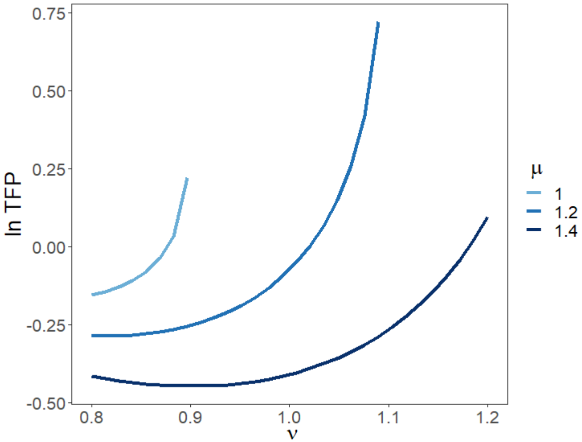
- TFP expression is nonlinear in ν

	Parameter	Value	Target
β	Discount rate	0.96	Real interest rate
δ	Depreciation rate	0.08	Office for National Statistics
ν	Variable RTS	0.99 - 1.05	ABS (authors' estimates)
μ	Markup	1.21 - 1.28	CMA (2022)
α	Capital share	0.25	ABS (authors' calculations)
ϑ	Pareto shape	10	Hopenhayn (2014)
κ	Entry cost	0.03	Match share inactive firms
ϕ	Overhead cost	0.2	Match share inactive firms

ν raises productivity

- Higher markups have a levels effect
- Weaken pass-through of RTS to TFP – mainly due to weaker selection decomposition

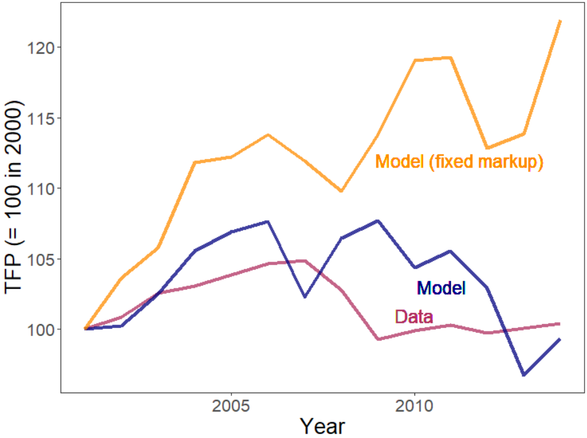
Figure 6 Effect of ν on TFP for different μ



Both ν and μ matter!

Calibrated Model

Figure 7 TFP Growth: Model vs Data



The TFP data series is from FRED.

Summing Up

- How can we understand rising returns to scale and stagnating productivity?
- Empirical results confirm returns to scale have increased whilst productivity has stagnated.
- Absent of markup increases productivity should've increased 20% over last two decades through RTS rises.
- However markup increases wipe out all these gains.
- RTS can increase through different sources – technologies that have reduced MC rather than increased FC seem more plausible.

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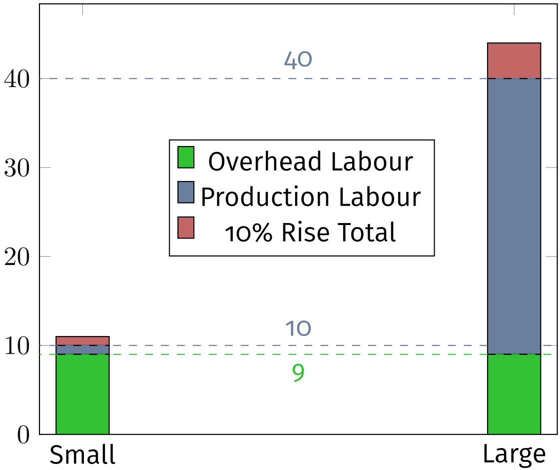
Returns to Scale and Firm Size (Production Side)

Intuition – Why do small firms have greater RTS?



- 10% rise in **total labour** raises **production labour** by 100% for small firm, just 13% for large firm.
- Small firm has greater returns to scale because output more responsive to inputs.

Pareto Productivity Distribution I

Productivity $A(\jmath)$ is a random draw on the unit interval $\jmath \in [0,1]$ using inverse transform sampling. The Pareto CDF is given by

$$F(A;\vartheta) = 1 - \left(\frac{h}{A}\right)^{\vartheta}; \quad A \geq h > 0 \quad \text{and} \quad \vartheta > 0.$$

If $\mathcal{J} \sim Uniform(0,1]$, then for $\jmath \in \mathcal{J}$, we have

$$1 - \left(\frac{h}{A}\right)^{\vartheta} = \jmath$$

Therefore

$$A(\jmath) = h(1-\jmath)^{-\frac{1}{\vartheta}}.$$

We set the scale parameter – which is the minimum possible value of A – to $h = 1$. Calibrations of the shape parameter (tail index) are set to match the firm size distribution, for example $\vartheta = 1.15$ in Barseghyan and DiCecio (2011) and $\vartheta = 1.06$ in Luttmer (2007) and $\vartheta = 6.10$ in Asturias, Hur, Kehoe, and Ruhl (2022).

Pareto Productivity Distribution II

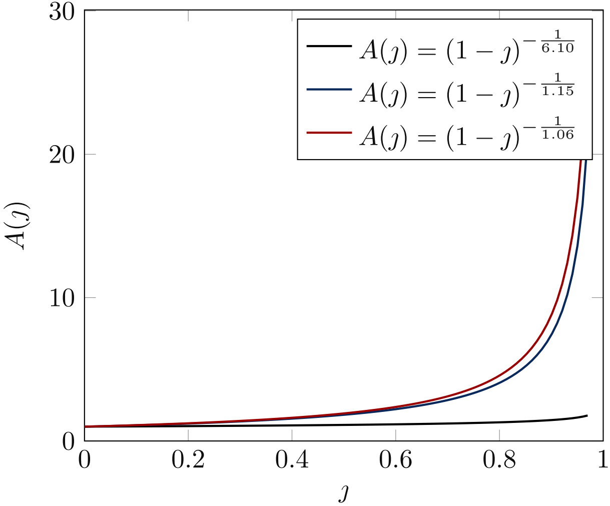
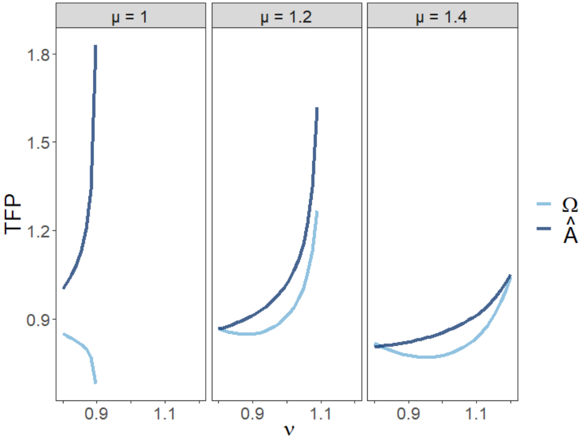


Figure 8 Productivity with Pareto Distribution, $h = 1, \vartheta = \{1.06, 1.15, 6.10\}$

Markups weaken the selection effect of higher RTS

- Higher markups weaken the selection channel which limits the productivity gains from higher RTS.
- Higher RTS increase productivity through greater selection but that channel is weakened when markups go up

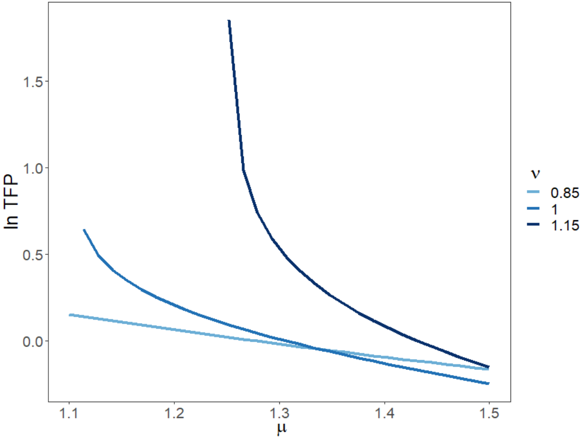
Figure 9 Effect of variable RTS on decomposed TFP for different markups



μ lowers productivity

- Higher markups reduce TFP (weaker selection)
- Rising markups are made worse by higher returns to scale

Figure 10 Effect of μ on TFP for different ν



Full Model

Household

- Household solves

$$
\begin{aligned}
\max_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t U(C_t), \quad \beta \in (0,1), \\
\text{s.t.} \quad & C_t + I_t = r_t K_t + w_t L^{\mathrm{s}} + \Pi_t + T_t \\
& I_t = K_{t+1} - (1-\delta) K_t.
\end{aligned}
$$

- $L^{\mathrm{s}} = 1$ normalise labour supply.
- T_t are entry costs that government rebates to households.
- Π_t are total profits. Revenue less factor payments less entry costs.
- Optimality condition is

$$
\left(\frac{C_{t+1}}{C_t}\right)^{\sigma} = \beta(r_{t+1} + (1-\delta)).
$$

Factor Market Equilibrium

- Intermediate goods producer solves

$$\pi_t(\jmath) = \max_{k_t(\jmath), \ell_t(\jmath)} p_t(\jmath) y_t(\jmath) - r_t k_t(\jmath) - w_t(\ell_t(\jmath) + \phi)$$

- Subject to production function and inverse demand.
- Results in factor market equilibrium:

$$\begin{aligned} \frac{r_t}{p_t(\jmath)} &= \frac{\nu}{\mu} \alpha \frac{y_t(\jmath)}{k_t(\jmath)} \\ \frac{w_t}{p_t(\jmath)} &= \frac{\nu}{\mu}(1-\alpha) \frac{y_t(\jmath)}{\ell_t(\jmath)}. \end{aligned}$$

- Real factor prices equal to marginal revenue products of input.
- Firms charge markup $\mu \in (1, \infty)$ of price over marginal cost.

Free Entry

- All firms die after one period.
- A firm only produces if it makes positive profits, hence firm value is given by
$$v_t(\jmath) = \max\{\pi_t(\jmath), 0\}.$$
- Free entry condition implies that the expected value of a firm equals the entry cost
$$\int_0^1 v_t(\jmath) d\jmath = \kappa.$$

Firm size ratio

- From factor market equilibrium and inverse demand function.
- For two firms $\imath$ and $\jmath$ the ratio of firm size equals the scaled productivity ratio:

$$\frac{p_t(\jmath)y_t(\jmath)}{p_t(\imath)y_t(\imath)} = \frac{k_t(\jmath)}{k_t(\imath)} = \frac{\ell_t(\jmath)}{\ell_t(\imath)} = \left[\frac{A_t(\jmath)}{A_t(\imath)}\right]^{\frac{1}{\mu-\nu}} \quad \forall \imath, \jmath,$$

Zero-Profit Productivity Threshold

- Given factor market equilibrium, profits are

$$\pi(\jmath) = \left(1 - \frac{\nu}{\mu}\right) p_t(\jmath) y_t(\jmath) - w_t \phi$$

- At productivity draw $J_t \in (0, 1)$ firm makes zero profit

$$\left(1 - \frac{\nu}{\mu}\right) p_t(J_t) y_t(J_t) - w_t \phi = 0.$$

- Productivity draw $\jmath \in (0, J_t)$ firm inactive; $\jmath \in (J_t, 1)$ firm active
- $J_t \uparrow$ stronger selection . $J_t \downarrow$ weaker selection.
- Expected profits conditional on surviving are

$$\mathbb{E}[\pi_t] = (1 - J_t) w_t (J_t) \phi \left(\frac{1}{1 - J_t} \int_{J_t}^{1} \left[\frac{A(\jmath)}{A(J_t)} \right]^{\frac{1}{\mu - \nu}} d\jmath - 1 \right)$$

Aggregation I

- Operating firms are a fraction of entering firms

$$N_t = E_t \int_{J_t}^{1} d\jmath = E_t(1 - J_t)$$

- J_t is the fraction of inactive firms ("exit") or *selection*.
- $1 - J_t = N/E$ is fraction of active firms ("survival").
- Aggregate factor inputs

$$K_t = E_t \int_{J_t}^{1} k_t(\jmath)\, d\jmath$$

$$L_t = E_t \int_{J_t}^{1} \ell_t(\jmath) + \phi\, d\jmath$$

Aggregation II

- Utilization u_t as the fraction of production labour in total labour:

$$u_t \equiv \frac{E_t \int_{J_t}^{1} \ell(\jmath) d\jmath}{L_t} \qquad 1 - u_t = \frac{N_t \phi}{L_t}.$$

- We can derive aggregate output

$$Y_t = TFP(J_t) K_t^{\alpha\nu} L_t^{1-\alpha\nu}$$

- TFP term captures aggregate productivity and can be decomposed into two terms $TFP_t = \Omega_t \times \hat{A}_t$:

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- If $\phi = 0$ then $u_t = 1$ and $TFP_t = \hat{A}_t$

Closing the model

- The resource constraint

$$Y_t = C_t + I_t$$

- Entry fees are rebated to households by the government

$$T_t = E_t \kappa$$

- Profits and labour market clearing

$$\Pi_t = \Pi_t^F$$
$$L_t = L_t^s$$